FACTORIZATION

Factorization: If a polynomial $p(x)$ can be expressed as $p(x) = g(x) \cdot h(x)$, then each of the polynomials $g(x)$ and $h(x)$ is called a factor of $p(x)$. The process of finding the factors is called factorization.

(a) Factorization of the Expression of the type $ka + kb + kc$.

Example

Factorize $5a - 5b + 5c$

Solution

$5a - 5b + 5c = 5(a - b + c)$

Example

Factorize $5a - 5b - 15c$

Solution

$5a - 5b - 15c = 5(a - b - 3c)$

(b) Factorization of the Expression of the type $ac + ad + bc + bd$

We can write $ac + ad + bc + bd$ as

$(ac + ad) + (bc + bd)$

$= a(c + d) + b(c + d)$

$= (a + b)(c + d)$

Example

Factorize $3x - 3a + xy - ay$

Solution

Regrouping the terms of given polynomial

$3x + xy - 3a - ay = x(3 + y) - a(3 + y)$

$= (3 + y)(x - a)$

(d) Factorization of the Expression of the type $a^2 - b^2$.

Example

Factorize $pqr + qr^2 - pr^2 - r^3$

Solution:

The given expression $= r(pq + qr - pr - r^2)$

$= r[(pq + qr) - pr - r^2]$  

$= r[q(p + r) - r(p + r)]$  

$= r(p + r)(q - r)$

(c) Factorization of the Expression of the type $a^2 \pm 2ab + b^2$.

We know that

(i) $a^2 + 2ab + b^2 = (a+b)^2 = (a+b)(a+b)$

(ii) $a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$

Example

Factorization $25x^2 + 16 + 40x$.

Solution:

$25x^2 + 40x + 16 = (5x)^2 + 2(5x)(4) + (4)^2$

$= (5x + 4)^2$

$= (5x + 4)(5x + 4)$

Example

Factorize $12x^2 - 36x + 27$

Solution:

$12x^2 - 36x + 27 = 3(4x^2 - 12x + 9)$

$= 3[(2x)^2 - 2(2x)(3) + (3)^2]$  

$= 3(2x - 3)^2$

$= 3(2x - 3)(2x - 3)$
(i) \(4x^2 -(2y-z)^2\)  
(ii) \(6x^4 - 96\)

**Solution**

(i) \(4x^2 -(2y-z)^2 = (2x)^2 -(2y-z)^2\)
   
   \[= [2x-(2y-z)][2x+(2y-z)]\]
   
   \[= (2x-2y+z)(2x+2y-z)\]

(ii) \(6x^4 - 96 = 6(x^4-16)\)
   
   \[= 6\left[(x^2)^2 - (4)^2\right]\]
   
   \[= 6(x^2-4)(x^2+4)\]
   
   \[= 6\left[(x)^2 - (2)^2\right](x^2+4)\]
   
   \[= 6(x-2)(x+2)(x^2+4)\]

(e) **Factorization of the Expression of the types** \(a^2 \pm 2ab + b^2 - c^2\).

We know that \(a^2 \pm 2ab + b^2 - c^2 = (a \pm b)^2 - (c)^2 = (a \pm b - c)(a \pm b + c)\)

**Example**

Factorize

(i) \(x^2 + 6x + 9 - 4y^2\)
(ii) \(1+2ab - a^2 - b^2\)

**Solution:**

(i) \(x^2 + 6x + 9 - 4y^2 = (x+3)^2 -(2y)^2\)
   
   \[= (x+3+2y)(x+3-2y)\]

(ii) \(1+2ab - a^2 - b^2 = 1 -(a^2 - 2ab + b^2)\)
   
   \[= (1)^2 -(a-b)^2\]
   
   \[= [1-(a-b)][1+(a-b)]\]
   
   \[= (1-a+b)(1+a-b)\]

**Exercise 5.1**

<table>
<thead>
<tr>
<th>Q.1</th>
<th>Factorize</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) (2abc-4abx+2abd)</td>
<td>(=3y(3x-4x^2+6y))</td>
</tr>
<tr>
<td>(=2ab(c-2x+d))</td>
<td>(iii) (-3x^2y-3x+9xy^2)</td>
</tr>
<tr>
<td>(ii) (9xy-12x^2y+18y^2)</td>
<td>(=-3x(xy+1-3y^2))</td>
</tr>
</tbody>
</table>
(iv) \[ 5ab^2c^3 - 10a^2b^3c + 20a^3bc^2 \]
\[ = 5abc(bc^2 - 2ab^2 + 4a^2c) \]
\[ = (12a+1)^2 \]
\[ = (12a+1)(12a+1) \]

(v) \[ 3x^3y(x-3y) - 7x^2y^2(x-3y) \]
\[ = (x-3y)(3x^3y - 7x^2y^2) \]
\[ = x^2y(x-3y)(3x-7y) \]
\[ \Rightarrow x^2y(x-3y)(3x-7y) \]

(vi) \[ 2xy^3(x^2+5) + 8xy^2(x^2+5) \]
\[ = (x^2+5)(2xy^3 + 8xy^2) \]
\[ = 2xy^2(x^2+5)(y+4) \]
\[ = (12a+1)^2 \]
\[ = (12a+1)(12a+1) \]

\[ \frac{a^2}{b^2} - 2 + \frac{b^2}{a^2} \]
\[ = \left( \frac{a}{b} \right)^2 - 2 \left( \frac{a}{b} \right) \left( \frac{b}{a} \right) + \left( \frac{b}{a} \right)^2 \]
\[ = \left( \frac{a}{b} - \frac{b}{a} \right)^2 \]
\[ = \left( \frac{a^2 - b^2}{ab} \right) \left( \frac{a^2 - b^2}{ab} \right) \]

(iii) \[ (x+y)^2 - 4z(x+y) + 49z^2 \]
\[ = (x+y)^2 - 2(x+y)(7z) + (7z)^2 \]
\[ = (x+y-7z)^2 \]
\[ = (x+y-7z)(x+y-7z) \]

(iv) \[ 12x^2 - 36x + 27 \]
\[ = 3(4x^2 - 12x + 9) \]
\[ = 3[(2x)^2 - 2(2x)(3) + (3)^2] \]
\[ = 3(2x-3)^2 \]
\[ = 3(2x-3)(2x-3) \]

Q.2
(i) \[ 5ax - 3ay - 5bx + 3by \]
\[ = 5ax - 5bx - 3ay + 3by \]
\[ = 5a(x-b) - 3y(a-b) \]
\[ = (a-b)(5x-3y) \]

(ii) \[ 3xy + 2y - 12x - 8 \]
\[ = 3xy - 12x + 2y - 8 \]
\[ = 3x(y-4) + 2(y-4) \]
\[ = (y-4)(3x+2) \]

(iii) \[ x^3 + 3xy^2 - 2x^2y - 6y^3 \]
\[ = x^3 - 2x^2y + 3xy^2 - 6y^3 \]
\[ = x^3 - 2y(x-3y) + 3y^2(3x) \]
\[ = (x-2y)(x^2 + 3y^2) \]

(iv) \[ (x^2 - y^2)z + (y^2 - z^2)x \]
\[ = x^2z - y^2z + y^2x - z^2x \]
\[ = x^2z - z^2x + y^2x - y^2z \]
\[ = xz(x-z) + y^2(x-z) \]
\[ = (x-z)(xz+y^2) \]

Q.3
(i) \[ 144a^2 + 24a + 1 \]
\[ = (12a)^2 + 2(12a)(1) + (1)^2 \]

(ii) \[ (x+y)^2 - 2(x+y)(7z) + (7z)^2 \]
\[ = (x+y)^2 - 2(x+y)(7z) + (7z)^2 \]
\[ = (x+y-7z)(x+y-7z) \]

(iii) \[ (x+y)^2 - 4z(x+y) + 49z^2 \]
\[ = (x+y)^2 - 2(x+y)(7z) + (7z)^2 \]
\[ = (x+y-7z)^2 \]
\[ = (x+y-7z)(x+y-7z) \]

(iv) \[ 12x^2 - 36x + 27 \]
\[ = 3(4x^2 - 12x + 9) \]
\[ = 3[(2x)^2 - 2(2x)(3) + (3)^2] \]
\[ = 3(2x-3)^2 \]
\[ = 3(2x-3)(2x-3) \]
(iii)  
\[128am^2 - 242an^2 = 2a(64m^2 - 121n^2) = 2a[(8m)^2 - (11n)^2] = 2a(8m + 11n)(8m - 11n)\]

(iv)  
\[3x - 243x^3 = 3x(1 - 81x^2) = 3x[(1)^2 - (9x)^2] = 3x(1 + 9x)(1 - 9x)\]

Q.5  
(i)  
\[x^2 - y^2 - 6y - 9 = x^2 - (y^2 + 6y + 9) = x^2 - [(y)^2 + 2(y)(3) + (3)^2] = (x)^2 - (y + 3)^2 = [(x) + (y + 3)][(x) - (y + 3)] = (x + y + 3)(x - y - 3)\]

(ii)  
\[x^2 - a^2 + 2a - 1 = x^2 - (a^2 - 2a + 1) = (x)^2 - (a - 1)^2 = [(x) + (a - 1)][(x) - (a - 1)] = (x + a - 1)(x - a + 1)\]

(iii)  
\[4x^2 - y^2 - 2y - 1 = 4x^2 - (y^2 + 2y + 1) = (2x)^2 - (y + 1)^2 = [(2x) + (y + 1)][(2x) - (y + 1)] = (2x + y + 1)(2x - y - 1)\]

(iv)  
\[x^2 - y^2 - 4x - 2y + 3 = x^2 - y^2 - 4x - 2y + 4 - 1 = x^2 - y^2 - 4x - 2y + 4 - 1\]

\[= x^2 - 4x + 4 - y^2 - 2y - 1 = (x)^2 - 2(x)(2) + (2)^2 - (y^2 + 2y + 1) = (x - 2)^2 - (y + 1)^2 = [(x - 2) + (y + 1)][(x - 2) - (y + 1)] = (x - 2 + y + 1)(x - 2 - y - 1) = (x + y - 1)(x - y - 3)\]

(v)  
\[25x^2 - 10x + 1 - 36z^2 = (5x)^2 - 2(5x)(1) + (1)^2 - (6z)^2 = (5x - 1)^2 - (6z)^2 = [(5x - 1) + (6z)][(5x - 1) - (6z)] = (5x - 1 + 6z)(5x - 1 - 6z) = (5x + 6z - 1)(5x - 6z - 1)\]

(vi)  
\[x^2 - y^2 - 4xz + 4z^2 = x^2 - 4xz + 4z^2 - y^2 = (x)^2 - 2(x)(2z) + (2z)^2 - (y)^2 = (x - 2z)^2 - (y)^2 = [(x - 2z) + (y)][(x - 2z) - (y)] = (x - 2z + y)(x - 2z - y)\]

(a) Factorization of the Expression of types \(a^4 + a^2b^2 + b^4\) or \(a^4 + 4b^4\)

Factorization of such types of expression is explained in the following examples.

Example

Factorize \(81x^4 + 36x^2y^2 + 16y^4\)

Solution

\[81x^4 + 36x^2y^2 + 16y^4 = (9x^2)^2 + 72x^2y^2 + (4y^2)^2 - 36x^2y^2 = (9x^2)^2 + (4y^2)^2 + 2(9x^2)(4y^2) - 36x^2y^2\]
\[
= (9x^2 + 4y^2)^2 - (6xy)^2 \\
= (9x^2 + 4y^2 + 6xy)(9x^2 + 4y^2 - 6xy) \\
= (9x^2 + 6xy + 4y^2)(9x^2 - 6xy + 4y^2)
\]

**Example**

Factorize \(9x^4 + 36y^4\)

**Solution:**

\[
9x^4 + 36y^4 = 9x^4 + 36y^4 + 36x^2y^2 - 36x^2y^2 \\
= (3x^2)^2 + 2(3x^2)(6y^2) + (6y^2)^2 - (6xy)^2 \\
= (3x^2 + 6y^2)^2 - (6xy)^2 \\
= (3x^2 + 6y^2 + 6xy)(3x^2 + 6y^2 - 6xy) \\
= (3x^2 + 6y^2 + 6xy)(3x^2 - 6xy + 6y^2)
\]

(b) **Factorization of the Expression of the type \(x^2 + px + q\).**

**Example**

Factorize (i) \(x^2 - 7x + 12\) 
(ii) \(x^2 + 15x - 36\)

**Solution:**

(i) \(x^2 - 7x + 12\)

From the factors of 12 the suitable pair of numbers is \(-3\) and \(-4\) since 

\((-3) + (-4) = -7\ and \ (-3)(-4) = 12\)

Hence \(x^2 - 7x + 12 = x^2 - 3x - 4x + 12\)

\(= x(x - 3) - 4(x - 3)\)

\(= (x - 3)(x - 4)\)

(ii) \(x^2 + 5x - 36\)

From the possible factors of 36, the suitable pair is 9 and \(-4\) because

\(9 + (-4) = 5\ and \ 9(-4) = -36\)

Hence \(x^2 + 5x - 36 = x^2 + 9x - 4x - 36\)

\(= x(x + 9) - 4(x + 9)\)

\(= (x + 9)(x - 4)\)

(c) **Factorization of the Expression of the type \(ax^2 + bx + c, a \neq 0\)**

**Example**

Factorize (i) \(9x^2 + 21x - 8\)

(ii) \(2x^2 - 8x - 42\)

(iii) \(10x^2 - 41xy + 21y^2\)

**Solution:**

(i) \(9x^2 + 21x - 8\)

In this case, on comparing with \(ax^2 + bx + c, ac = (9)(-8) = -72\).

From the possible factors of 72 the suitable pair of numbers (with proper sign) is 24 and \(-3\) whose 

Sum = 24 + \((-3) = 21\), (the coefficient of \(x\))

And their product = (24)(-3) = -72 = ac

Hence \(9x^2 + 21x - 8\)

\(= 9x^2 + 24x - 3x - 8\)

\(= 3x(3x + 8) - 1(3x + 8)\)

\(= (3x + 8)(3x - 1)\)

(ii) \(2x^2 - 8x - 42 = 2(x^2 - 4x - 21)\)

Comparing \(x^2 - 4x - 21 with ax^2 + bx + c\)

We have \(ac = (+1)(-21) = -21\)

From the possible factors of 21 the suitable pair of numbers is \(-7\) and \(+3\) whose 

Sum = \(-7 + 3 = -4\) and product = \((-7)(3) = -21\)

Hence \(x^2 - 4x - 21\)

\(= x^2 + 3x - 7x - 21\)

\(= x(x + 3) - 7(x + 3)\)
\[(x + 3)(x - 7)\]

Hence \[2x^2 - 8x - 42 = 2(x^2 - 4x - 21) = 2(x + 3)(x - 7)\]

(iii) \[10x^2 - 41xy + 21y^2\]
Here \(ac = (10)(21) = 210\)
Two suitable factors of 210 are \(-35\) and \(-6\).
Their sum = \(-35 - 6 = -41\)
And product = \((-35)(-6) = 210\)
Hence \[10x^2 - 41xy + 21y^2\]
\[= 10x^2 - 35xy - 6xy + 21y^2\]
\[= 5x(2x - 7y) - 3y(2x - 7y)\]
\[= (2x - 7y)(5x - 3y)\]

(d) Factorization of the following types of Expressions.
\[(ax^2 + b + c)(ax^2 + bx + d) + k\]
\[(x + a)(x + b)(x + c)(x + d) + k\]
\[(x + a)(x + b)(x + c)(x + d) + kx^2\]

Example
Factorize \((x^2 - 4x - 5)(x^2 - 4x - 12) - 144\)
Solution:
\[(x^2 - 4x - 5)(x^2 - 4x - 12) - 144\]
Let \(y = x^2 - 4x\). Then
\[(y - 5)(y - 12) - 144 = y^2 - 17y + 60 - 144 \]
\[= y^2 - 17y - 84\]
\[= y^2 - 21y + 4y - 84\]
\[= y(y - 21) + 4(y - 21)\]
\[= (y - 21)(y + 4)\]
\[= (x^2 - 4x - 21)(x^2 - 4x + 4) \quad (Since \ y = x^2 - 4x)\]
\[= (x^2 - 7x + 3x - 21)[(x)^2 - 2(x)(2) + (2)^2]\]
\[= [x(x - 7) + 3(x - 7)](x - 2)^2\]
\[= (x - 7)(x + 3)(x - 2)(x - 2)\]
Example

Factorize

\((x+1)(x+2)(x+3)(x+4)-120\)

Solution:
We observe that \(1+4=2+3\).
It suggests that we rewrite the given expression as

\[[(x+1)(x+4)][(x+2)(x+3)]-120\]

\((x^2+5x+4)(x^2+5x+6)-120\)

Let \(x^2+5x=y\), then
We get \((y+4)(y+6)-120\)
\[=y^2+10y+24-120\]
\[=y^2+10y-96\]
\[=y^2+16y-6y-96\]
\[=y(y+16)-6(y+16)\]
\[=(y+16)(y-6)\]
\[=(x^2+5x+16)(x^2+5x-6)\text{ (since } y=x^2+5x)\]
\[=(x^2+5x+16)[x^2+6x-x-6]\]
\[=(x^2+5x+16)[(x+6)-1(x+6)]\]
\[=(x^2+5x+16)(x+6)(x-1)\]

Example

Factorize \((x^2-5x+6)(x^2+5x+6)-2x^2\)

Solution:

\[(x^2-5x+6)(x^2+5x+6)-2x^2\]
\[=\left[x^2-3x-2x+6\right][x^2+3x+2x+6]-2x^2\]
\[=[x(x-3)-2(x-3)][x(x+3)+2(x+3)]-2x^2\]
\[=\left[(x-3)(x-2)\right]\left[(x+3)(x+2)\right]-2x^2\]
\[=\left[(x-2)(x+2)\right]\left[(x-3)(x+3)\right]-2x^2\]
\[=(x^2-4)(x^2-9)-2x^2\]

\[=x^4-13x^2+36-2x^2\]
\[=x^4-15x^2+36\]
\[=x^4-12x^2-3x^2+36\]
\[=x^2(x^2-12)-3(x^2-12)\]
\[=(x^2-12)(x^2-3)\]
\[=\left[(x^2-2\sqrt{3})^2\right]\left[(x^2-(\sqrt{3})^2\right]\]
\[=(x-2\sqrt{3})(x+2\sqrt{3})(x-\sqrt{3})(x+\sqrt{3})\]

(e) Factorization of Expressions of the following Types

\[a^3+3a^2b+3ab^2+b^3\]
\[a^3-3a^2b+3ab^2-b^3\]

Example:

Factorize \(x^2-8y^3-6x^2y+12xy^2\)

Solution:

\[x^2-8y^3-6x^2y+12xy^2\]
\[=(x)^3-(2y)^3-3(x)^2(2y)+3(x)(2y)^2\]
\[=(x)^3-3(x)^2(2y)+3(x)(2y)^2-(2y)^3\]
\[=(x-2y)^3\]
\[=(x-2y)(x-2y)(x-2y)\]

(d) Factorization of Expressions of the following types \(a^3 \pm b^3\)

We recall the formulas,

\[a^3+b^3=(a+b)(a^2-ab+b^2)\]
\[a^3-b^3=(a-b)(a^2+ab+b^2)\]

Example

Factorize \(27x^3+64y^3\)

Solution:

\[27x^3+64y^3=(3x)^3+(4y)^3\]
\[=(3x+4y)(9x^2-12xy+16y^2)\]
Example

Factorize $1-125x^3$

Solution

$1-25x^3 = (l)^3 - (5x)^3$

$$= (1-5x)[(l)^2 + (1)(5x) + (5x)^2]$$
$$= (1-5x)(1+5x+25x^2)$$

Exercise 5.2

Q.1 Factorize

(i) $x^4 + \frac{1}{x^4} - 3$

$$= x^4 + \frac{1}{x^4} - 2 - 1$$

$$= (x^2 + \frac{1}{x^2})^2 - 2(x^2)(\frac{1}{x^2}) - 1$$

$$= \left(x^2 - \frac{1}{x^2} + 1\right)\left(x^2 - \frac{1}{x^2} - 1\right)$$

(ii) $3x^4 + 12y^4$

$$= 3\left(x^4 + 4y^4\right)$$

$$= 3\left[(x^2)^2 + 2(x^2)(2y^2) - (2y)^2\right]$$

$$= 3\left[(x^2 + 2y^2)^2 - (2xy)^2\right]$$

$$= 3(x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)$$

$$= 3(x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)$$

(iii) $a^8 + 3a^2b^2 + 4b^4$

$$= a^8 + 4a^2b^2 + 4b^4 - a^2b^2$$

$$= (a^8 + 2(a^2)(2b^2) + (2b^2)^2 - a^2b^2)$$

$$= (a^2 + 2b^2)^2 - (ab)^2$$

$$= (a^2 + 2b^2 + ab)(a^2 + 2b^2 - ab)$$

$$= (a^2 + ab + 2b^2)(a^2 - ab + 2b^2)$$

(iv) $4x^4 + 81$

$$= (2x^2)^2 + (9)^2 + 2(2x^2)(9) - 36x^2$$

$$= (2x^2 + 9)^2 - (6x)^2$$

$$= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$$

$$= (2x^2 + 9 + 6x)(2x^2 - 6x + 9)$$

(v) $x^4 + x^2 + 25$

$$= (x^2)^2 + 2(x^2)(5) + (5)^2 - 9x^2$$

$$= (x^2 + 5)^2 - (3x)^2$$

$$= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$$

$$= (x^2 + 3x + 5)(x^2 - 3x + 5)$$

(vi) $x^4 + 4x^2 + 16$

$$= (x^2)^2 + 2(x^2)(4) + (4)^2 - 4x^2$$

$$= (x^2 + 4)^2 - (2x)^2$$

$$= (x^2 + 4 + 2x)(x^2 + 4 - 2x)$$

$$= (x^2 + 2x + 4)(x^2 - 2x + 4)$$

Q.2

(i) $x^2 + 14x + 48$

$$= x^2 + 6x + 8x + 48$$

$$= x(x + 6) + 8(x + 6)$$

$$= (x + 6)(x + 8)$$

(ii) $x^2 - 21x + 108$

$$= x^2 - 9x - 12x + 108$$

$$= x(x - 9) - 12(x - 9)$$
= (x - 9) (x - 12)
(iii) \( x^2 - 11x - 42 \)
= \( x^2 + 3x - 14x - 42 \)
= \( x(x + 3) - 14(x + 3) \)
= \( (x + 3)(x - 14) \)
(iv) \( x^2 + x - 132 \)
= \( x^2 + 12x - 11x - 132 \)
= \( x(x + 12) - 11(x + 12) \)
= \( (x + 12)(x - 11) \)

Q.3
(i) \( 4x^2 + 12x + 5 \)
= \( 4x^2 + 2x + 10x + 5 \)
= \( 2x(2x + 1) + 5(2x + 1) \)
= \( (2x + 1)(2x + 5) \)

(ii) \( 30x^2 + 7x - 15 \)
= \( 30x^2 + 25x - 18x - 15 \)
= \( 5x(6x + 5) - 3(6x + 5) \)
= \( (6x + 5)(5x - 3) \)

(iii) \( 24x^2 - 65x + 21 \)
= \( 24x^2 - 56x - 9x + 21 \)
= \( 8x(3x - 7) - 3(3x - 7) \)
= \( (3x - 7)(8x - 3) \)

(iv) \( 5x^2 - 16x - 21 \)
= \( 5x^2 + 5x - 21x - 21 \)
= \( 5x(x + 1) - 21(x + 1) \)
= \( (x + 1)(5x - 21) \)

(v) \( 4x^2 - 17xy + 4y^2 \)
= \( 4x^2 - 16xy - xy + 4y^2 \)
= \( 4x(x - 4y) - y(x - 4y) \)
= \( (x - 4y)(4x - y) \)

(vi) \( 3x^2 - 38xy - 13y^2 \)
= \( 3x^2 - 39xy + xy - 13y^2 \)
= \( 3x(x - 13y) + y(x - 13y) \)
= \( (x - 13y)(3x + y) \)

(vii) \( 5x^2 + 33xy - 14y^2 \)
= \( 5x^2 + 35xy - 2xy - 14y^2 \)
= \( 5x(x + 7y) - 2y(x + 7y) \)
= \( (x + 7y)(5x - 2y) \)

(viii) \( \left( 5x - \frac{1}{x} \right)^2 + 4 \left( 5x - \frac{1}{x} \right) + 4 \)
= \( \left( 5x - \frac{1}{x} \right)^2 + 2 \left( 5x - \frac{1}{x} \right)(2) + (2)^2 \)
= \( \left( 5x - \frac{1}{x} + 2 \right)^2 \)
= \( \left( 5x - \frac{1}{x} + 2 \right) \left( 5x - \frac{1}{x} + 2 \right) \)

Q.4
(i) \( (x^2 + 5x + 4)(x^2 + 5x + 6) - 3 \)
Let \( x^2 + 5x = y \)
then
\( (x^2 + 5x + 4)(x^2 + 5x + 6) - 3 \)
= \( (y + 4)(y + 6) - 3 \)
= \( y^2 + 4y + 6y + 24 - 3 \)
= \( y^2 + 10y + 21 \)
= \( y^2 + 3y + 7y + 21 \)
= \( y(y + 3) + 7(y + 3) \)
= \( (y + 3)(y + 7) \)

Putting value of \( y \)
\( = (x^2 + 5x + 3)(x^2 + 5x + 7) \)

(ii) \( (x^2 - 4x)(x^2 - 4x - 1) - 20 \)
Let \( x^2 - 4x = y \)
then
\[
(x^2 - 4x)(x^2 - 4x - 1) - 20
= y(y - 1) - 20
= y^2 - y - 20
= y^2 + 4y - 5y - 20
= y(y + 4) - 5(y + 4)
= (y + 4)(y - 5)
\]

Putting value of \( y \)
\[
= (x^2 - 4x + 4)(x^2 - 4x - 5)
= [(x^2 - 2x + 2)^2] [x^2 + x - 5x - 5]
= (x - 2)^2 [x(x + 1) - 5(x + 1)]
= (x - 2)^2 (x + 1)(x - 5)
\]

(iii) \( (x + 2)(x + 3)(x + 4)(x + 5) - 15 \)
\[
= [(x + 2)(x + 5)][(x + 3)(x + 4)] - 15
= (x^2 + 2x + 5x + 10)(x^2 + 3x + 4x + 12) - 15
= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15
\]

Let \( x^2 + 7x = y \)
\[
= (y + 10)(y + 12) - 15
= y^2 + 10y + 12y + 120 - 15
= y^2 + 22y + 105
= y^2 + 7y + 15y + 105
= y(y + 7) + 15(y + 5)
= (y + 7)(y + 15)
\]

Putting value of \( y' \)
\[
(x + 2 + 7x + 7)(x^2 + 7x + 15)
\]

(iv) \( (x + 4)(x - 5)(x + 6)(x - 7) - 504 \)
\[
= (x^2 + 4x - 5x - 20)(x^2 + 6x - 7x - 42) - 504
= (x^2 - x - 20)(x^2 - x - 42) - 504
\]

Let \( x^2 - x = y \)
\[
= (y - 20)(y - 42) - 504
= y^2 - 20y - 42y + 840 - 504
= y^2 - 62y + 336
= y^2 - 6y - 56y + 336
= y(y - 6) - 56(y - 6)
= (y - 6)(y - 56)
\]

Putting value of \( y' \)
\[
= (x^2 - x - 6)(x^2 - x - 56)
= (x^2 + 2x - 3x - 6)(x^2 + 7x + 8x - 56)
= [(x + 2) - 3(x + 2)][x(x + 7) - 8(x + 7)]
= (x + 2)(x - 3)(x + 7)(x - 8)
\]

(v) \( (x + 1)(x + 2)(x + 3)(x + 6) - 3x^2 \)
\[
= (x + 1)(x + 6)(x + 2)(x + 3) - 3x^2
= (x^2 + 6x + 6)(x^2 + 2x + 3x + 6) - 3x^2
= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2
= \frac{x^2}{x^2} [(x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2]
= x^2 \left[ \left( x + \frac{6}{x} + 7 \right) \left( x + \frac{6}{x} + 5 \right) - 3 \right]
\]

Let \( x + \frac{6}{x} = y \)
\[
= x^2 [(y + 7)(y + 5) - 3]
= x^2 (y^2 + 7y + 5y + 35 - 3)
= x^2 (y^2 + 12y + 32)
= x^2 (y^2 + 4y + 8y + 32)
= x^2 [y(y + 4) + 8(y + 4)]
= x^2 (y + 4)(y + 8)
Putting value of \( y \)

\[
\begin{align*}
&= x^2 \left( \frac{x+6}{x} + 4 \right) \left( \frac{x+6}{x} + 8 \right) \\
&= x^2 \left( \frac{x^2+4x+6}{x} \right) \left( \frac{x^2+8x+6}{x} \right) \\
&= (x^2+4x+6)(x^2+8x+6) \\
&= (x^2+4x+6)(x^2+8x+6)
\end{align*}
\]

Q. 5

(i) \( x^3 + 48x - 12x^2 - 64 \)

\[
= x^3 - 12x^2 + 48x - 64 \\
= (x^3 - 3x^2)(4 - 3(4) - (4)^3) \\
= (x-4)(x-4)(x-4)
\]

(ii) \( 8x^3 + 60x^2 + 150x + 125 \)

\[
= (2x^3 + 3(2x)^2 + 3(2)(5x)^2 + 5)(2x + 5) \\
= (2x + 5)(2x + 5)(2x + 5)
\]

(iii) \( x^3 - 18x^2 + 108x - 216 \)

\[
= (x^3 - 3x^2)(6) + 3(x)(6)^2 - (6)^3 \\
= (x - 6)(x - 6)(x - 6)
\]

(iv) \( 8x^3 - 125y^3 - 60x^2y + 150xy^2 \)

\[
= 8x^3 - 60x^2y + 150xy^2 - 125y^3 \\
= (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3 \\
= (2x - 5y)^3 \\
= (2x - 5y)(2x - 5y)(2x - 5y)
\]

Q. 6

(i) \( 27 + 8x^3 \)

\[
= (3)^3 + (2x)^3 \\
= (3 + 2x) \left[ (3)^2 - (3)(2x) + (2x)^2 \right] \\
= (3 + 2x)(9 - 6x + 4x^2)
\]

or \( = (2x + 3)(4x^2 - 6x + 9) \)

(ii) \( 125x^2 - 216y^3 \)

\[
= (5x)^3 - (6y)^3 \\
= (5x - 6y)(5x + 6y) \\
= (5x - 6y)(25x^2 + 30xy + 36y^2)
\]

(iii) \( 64x^3 + 27y^3 \)

\[
= (4x)^3 + (3y)^3 \\
= (4x + 3y) \left[ (4x)^2 - (4x)(3y) + (3y)^2 \right] \\
= (4x + 3y)(16x^2 - 12xy + 9y^2)
\]

(iv) \( 8x^3 + 125y^3 \)

\[
= (2x)^3 + (5y)^3 \\
= (2x + 5y) \left[ (2x)^2 - (2x)(5y) + (5y)^2 \right] \\
= (2x + 5y)(4x^2 - 10xy + 25y^2)
\]

**Remainder Theorem**

If a polynomial \( p(x) \) is divided by a linear divisor \( (x - a) \), then the remainder is \( p(a) \).

**Proof**

Let \( q(x) \) be the quotient obtained after dividing \( p(x) \) by \( (x - a) \). But the divisor \( (x - a) \) is linear. So the remainder must be of degree zero i.e., a non-zero constant, say \( R \). Consequently, by division Algorithm we may write.

\[
p(x) = (x - a)q(x) + R
\]

This is an identity in \( x \) and so is true for all real numbers \( x \). In particular, it is true for \( x = a \). Therefore,

\[
p(a) = (a - a)q(a) + R = 0 + R = R
\]

i.e., \( p(a) = R \) the remainder.

Hence the theorem.

**Note:** Similarly, if the divisor is \( (ax - b) \), we have

\[
p(x) = (ax - b)q(x) + R
\]
Substituting \( x = \frac{b}{a} \) so that \( ax - b = 0 \), we obtain

\[
p\left(\frac{b}{a}\right) = 0, \quad q\left(\frac{b}{a}\right) + R = 0 + R = R
\]

Thus if the divisor is linear, the above theorem provides an efficient way of finding the remainder without being involved in the process of long division.

**To find remainder (without dividing) when a polynomial is divided by a Linear Polynomial**

**Example**

Find the remainder when

\( 9x^2 - 6x + 2 \) is divided by

(i) \( x - 3 \)  \hspace{1cm} (ii) \( x + 3 \)

(iii) \( 3x + 1 \)  \hspace{1cm} (iv) \( x \)

**Solution:**

Let \( p(x) = 9x^2 - 6x + 2 \)

(i) When \( p(x) \) is divided by \( x - 3 \), by Remainder Theorem, the remainder is:

\[
R = p(3) = 9(3)^2 - 6(3) + 2 = 65
\]

(ii) When \( p(x) \) is divided by \( x + 3 = x - (-3) \), the remainder is

\[
R = p(-3) = 9(-3)^2 - 6(-3) + 2 = 9(9) + 18 + 2 = 81 + 20 = 101
\]

(iii) When \( p(x) \) is divided by \( 3x + 1 \), the remainder is

\[
R = p\left(-\frac{1}{3}\right) = 9\left(-\frac{1}{3}\right)^2 - 6\left(-\frac{1}{3}\right) + 2 = 5
\]

(iv) When \( p(x) \) is divided by \( x \), the remainder is

\[
R = p(0) = 9(0)^2 - 6(0) + 2 = 2
\]

**Example**

Find the value of \( k \) is the expression \( x^3 + kx^2 + 3x - 4 \) leaves a remainder of \(-2\) when divided by \( x + 2 \).

**Solution:**

Let \( p(x) = x^3 + kx^2 + 3x - 4 \).

By the remainder Theorem, when \( p(x) \) is divided by \( x + 2 = x - (-2) \), the remainder is:

\[
p(-2) = (-2)^3 + k(-2)^2 + 3(-2) - 4
\]

\[
= -8 + 4k - 6 - 4
\]

\[
= 4k - 18
\]

By the given condition, we have

\[
p(-2) = -2 \quad \Rightarrow \quad 4k - 18 = -2
\]

\[
\Rightarrow \quad k = 4
\]

**5.2.3 Zero of a polynomial**

If a specific number \( x = a \) is substituted for a variable \( x \) in a polynomial \( p(x) \) so that the value \( p(a) \) is zero, then \( x = a \) is called a zero of the polynomial \( p(x) \).

**Factor Theorem**

The polynomial \( (x - a) \) is a factor of the polynomial \( p(x) \) if and only if \( p(a) = 0 \).

**Proof:**

Let \( q(x) \) be the quotient and \( R \) the remainder when a polynomial \( p(x) \) is divided by \( (x - a) \). Then by division Algorithm,

\[
p(x) = (x - a)q(x) + R
\]

By the Remainder Theorem, \( R = p(a) \).
Hence \( p(x) = (x-a)q(x) + p(a) \)

(i) Now if \( p(a) = 0 \), then
\[ p(x) = (x-a)q(x) \]
i.e., \( (x-a) \) is a factor of \( p(x) \).

(ii) Conversely, if \( (x-a) \) is a factor of \( p(x) \), then the remainder upon dividing \( p(x) \) by \( (x-a) \) must be zero i.e., \( p(a) = 0 \).

**Example**

Determine if \( (x-2) \) is a factor of \( x^3 - 4x^2 + 3x + 2 \).

**Solution:**

Let
\[ p(x) = x^3 - 4x^2 + 3x + 2 \]
Then the remainder for \( (x-2) \) is:
\[ p(2) = (2)^3 - 4(2)^2 + 3(2) + 2 \]
\[ = 8 - 16 + 6 + 2 = 0 \]

Hence by Factor Theorem, \( (x-2) \) is a factor of the polynomial \( p(x) \).

**Example**

Find a polynomial \( p(x) \) of degree 3 that has 2, -1, and 3 as zeros (i.e., roots).

**Solution:**

Since \( x = 2, -1, 3 \) are roots of \( p(x) = 0 \).

So by Factor theorem \( (x-2), (x+1) \) and \( (x-3) \) are the factors of \( p(x) \).

Thus \( p(x) = a(x-2)(x+1)(x-3) \)

Where any non-zero value can be assigned to \( a \).

Taking \( a = 1 \), we get
\[ p(x) = (x-2)(x+1)(x-3) \]
\[ = x^3 - 4x^2 + x + 6 \] as the required polynomial.

**Exercise 5.3**

Q.1 Use the remainder theorem to find the remainder, when.
(i) \( 3x^3 - 10x^2 + 13x - 6 \) is divided by \( (x-2) \)

**Sol:**

Let \( P(x) = 3x^3 - 10x^2 + 13x - 6 \) when \( P(x) \) is divided by \( x - 2 \) by remainder theorem, the remainder is:
\[ R = P\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right) + 3 \]
\[ = 4\left(\frac{1}{8}\right) - 2 + 3 \]
\[ = \frac{1}{2} + 1 \]
\[ = \frac{1 + 2}{2} \]

(ii) \( 4x^3 - 4x + 3 \) is divided by \( (2x-1) \)

Sol:

Let \( P(x) = 4x^3 - 4x + 3 \) when \( P(x) \) is divided by \( 2x - 1 \) by remainder theorem, the remainder is:
\[ R = P\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right) + 3 \]
\[ = 4\left(\frac{1}{8}\right) - 2 + 3 \]
\[ = \frac{1}{2} + 1 \]
\[ = \frac{1 + 2}{2} \]
\[ R = \frac{3}{2} \]

(iii) \( 6x^4 + 2x^3 - x + 2 \) is divided by \( x + 2 \)

**Sol:**
Let \( P(x) = 6x^4 + 2x^3 - x + 2 \) when \( P(x) \) is divided by \( x + 2 \) by remainder theorem, the remainder is

\[
R = P(-2) = 6(-2)^4 + 2(-2)^3 - (-2) + 2 \\
= 6(16) + 2(-8) + 2 + 2 \\
= 96 - 16 + 4 \\
= 80 + 4 \\
R = 84
\]

(iv) \( (2x - 1)^3 + 6(3 + 4x)^2 - 10 \) is divided by \( 2x + 1 \)

**Sol:**
Let \( p(x) = (2x - 1)^3 + 6(3 + 4x)^2 - 10 \) when \( P(x) \) is divided by \( 2x + 1 \) by remainder theorem, then remainder is

\[
R = p \left( \frac{-1}{2} \right) = \left[ 2 \left( \frac{-1}{2} \right) - 1 \right]^3 + 6 \left[ 3 + 4 \left( \frac{-1}{2} \right) \right]^2 - 10 \\
= (-1-1)^3 + 6(3-2)^2 - 10 \\
= (-2)^3 + 6(1)^2 - 10 \\
= -8 + 6 - 10 \\
= -12
\]

(v) \( x^3 - 3x^2 + 4x - 14 \) is divided by \( x + 2 \)

**Sol:**
Let \( P(x) = x^3 - 3x^2 + 4x - 14 \) when \( P(x) \) is divided by \( x + 2 \) by remainder theorem, then remainder is

\[
R = P(-2) = (-2)^3 - 3(-2)^2 + 4(-2) - 14 \\
= -8 - 3(4) - 8 - 14 \\
= -8 - 12 - 8 - 14 \\
= -42
\]

Q.2.

(i) If \( (x+2) \) is a factor of \( 3x^2 - 4kx - 4k^2 \), then find the value(s) of \( k \).

**Sol:**
Let \( P(x) = 3x^2 - 4kx - 4k^2 \)

As given that \( x + 2 \) is a factor of \( P(x) \), so

\[ R = 0 \]

i.e. \( P(-2) = 0 \)

So \( 3(-2)^2 - 4k(-2) - 4k^2 = 0 \)

\( 12 + 8k - 4k^2 = 0 \)

Dividing by 4

\( 3 + 2k - k^2 = 0 \)

\( 3 + 3k - k^2 = 0 \)

\( 3(1 + k) - k(1 + k) = 0 \)

\( (1 + k)(3 - k) = 0 \)

\( \Rightarrow 1 + k = 0 \) or \( 3 - k = 0 \)

\( \Rightarrow k = -1 \) or \( k = 3 \)

(ii) If \( (x - 1) \) is factor of \( x^3 - kx^2 + 11x - 6 \) then find the value of \( k \).

**Sol:**
\( P(x) = x^3 - kx^2 + 11x - 6 \)

As given that \( x - 1 \) is a factor of \( P(x) \), so

\[ R = 0 \]

\[ P(1) = 0 \]

\( (1)^3 - k(1)^2 + 11(1) - 6 = 0 \)

\( 1 - k + 11 - 6 = 0 \)

\( 6 - k = 0 \)

\( \Rightarrow k = 6 \)

Q.3 Without actual long division determine whether

(i) \( (x - 2) \) and \( (x - 3) \) are factors of \( P(x) = x^3 - 12x^2 + 44x - 48 \)
Sol:
\[ P(x) = x^3 - 12x^2 + 44x - 48 \]
Taking \( x = 2 \)
\[ R = P(2) \]
\[ = (2)^3 - 12(2)^2 + 44(2) - 48 \]
\[ = 8 - 12(4) + 88 - 48 \]
\[ = 8 - 48 + 88 - 48 \]
\[ = 0 \]
As the remainder is zero, so \( (x - 2) \) is a factor of \( P(x) \)
Now \( P(x) = x^3 - 12x^2 + 44x - 48 \)
Taking \( x = 3 \)
\[ R = P(3) \]
\[ = (3)^3 - 12(3)^2 + 44(3) - 48 \]
\[ = 27 - 12(9) + 132 - 48 \]
\[ = 27 - 108 + 132 - 48 \]
\[ = 3 \neq 0 \]
As the remainder is not equal to zero, so \( (x - 3) \) is not a factor of \( P(x) \).

(ii) \( (x - 2) \), \( (x + 3) \) and \( (x - 4) \) are factors of \( q(x) = x^3 + 2x^2 - 5x - 6 \)

Sol:
\[ q(x) = x^3 + 2x^2 - 5x - 6 \]
Taking \( x = 2 \)
\[ R = q(2) = (2)^3 + 2(2)^2 - 5(2) - 6 \]
\[ = 8 + 2(4) - 10 - 6 \]
\[ R = 0 \]
As the remainder is zero
so \( (x - 2) \) is a factor of \( P(x) \)
Now \( q(x) = x^3 + 2x^2 - 5x - 6 \)
Taking \( x = 4 \)
\[ R = q(4) \]
\[ = (4)^3 + 2(4)^2 - 5(4) - 6 \]
\[ = 64 + 2(16) - 20 - 6 \]
\[ = 64 + 32 - 20 - 6 \]
\[ = 70 \neq 0 \]
As the remainder is not equal to zero, so \( x - 4 \) is not a factor of \( P(x) \).

Q.4 For what value of \( m \) is the polynomial \( P(x) = 4x^3 - 7x^2 + 6x - 3m \)

exactly divisible by \( x + 2 \)?

Sol:
\[ m = ? \]
\[ P(x) = 4x^3 - 7x^2 + 6x - 3m \]
Taking \( x = 2 \)
As \( p(x) \) is exactly divisible by \( (x + 2) \), so
\[ R = 0 \]
\[ P(-2) = 0 \]
\[ 4(-2)^3 - 7(-2)^2 + 6(-2) - 3m = 0 \]
\[ 4(-8) - 7(4) - 12 - 3m = 0 \]
\[ -32 - 28 - 12 - 3m = 0 \]
\[ -3m = 72 \]
\[
m = \frac{72}{-3} = -24
\]

**Q.5** Determine the value of \(k\) if
\[
P(x) = kx^3 + 4x^2 + 3x - 4\]
and
\[
q(x) = x^3 - 4x + k.
\]
Leaves the same remainder when divided by \(x - 3\).

**Sol:**

\[K = ?\]

When \(p(x)\) is divided by \((x - 3)\) by remainder theorem then remainder is
\[
R_1 = p(3)
\]
\[
= k(3)^3 + 4(3)^2 + 3(3) - 4
\]
\[
= 27k + 36 + 9 - 4
\]
\[
= 27k + 41
\]

When \(q(x)\) is divided by \((x - 3)\) by remainder theorem then remainder is
\[
R_2 = q(3)
\]
\[
q(x) = x^3 - 4x + k
\]
\[
= (3)^3 - 4(3) + k
\]
\[
= 27 - 12 + k
\]
\[
= 15 + k
\]

As given that when \(P(x)\) and \(q(x)\) are divided by \(x - 3\), then remainder is same, so
\[
R_1 = R_2
\]
\[
27k + 41 = 15 + k
\]
\[
27k - k = 15 - 41
\]
\[
26k = -26
\]
\[
k = -1
\]

**Q.6**

The remainder of dividing the polynomial
\[
P(x) = x^3 + ax^2 + 7\]
by \((x + 1)\) is \(2b\).

Calculate the value of \('a'\) and \('b'\) if this expression leaves a remainder of \((b + 5)\) on being divided by \((x - 2)\).

**Sol:**

\[
P(x) = x^3 + ax^2 + 7
\]

The remainder by dividing
\[
P(x) by \(x + 1\) is 2b, so
\]
\[
P(-1) = 2b
\]
\[
(1) + a(-1)^2 + 7 = 2b
\]
\[
1 + a + 7 = 2b
\]
\[
a + 6 = 2b
\]
\[
a - 2b = -6 \ldots \ldots (i)
\]

Taking \(x - 2\)

The remainder by dividing
\[
P(x) by (x - 2) is (b + 5), so
\]
\[
P(2) = b + 5
\]
\[
(2)^3 + a(2)^2 + 7 = b + 5
\]
\[
8 + 4a + 7 = b + 5
\]
\[
4a + 15 = b + 5
\]
\[
4a - b = 5 - 15
\]
\[
4a - b = -10 \ldots \ldots (ii)
\]

Multiplying \((ii)\) by 2
\[
8a - 2b = -20 \ldots \ldots (iii)
\]

By Subtracting \((iii)\) from \((i)\)
\[
a - 2b = -6
\]
\[
8a + 2b = \pm 20
\]
\[
-7a = 14
\]
\[
a = -2
\]

Putting \((i)\)
\[ a - 2b = -6 \]
\[ -2 - 2b = -6 \]
\[ -2b = -6 + 2 \]
\[ -2b = -4 \]
\[ b = 2 \]

Q.7 The polynomial \( x^3 + \ell x^2 + mx + 24 \) has a factor \( (x + 4) \) and it leaves a remainder of 36 when divided by \( (x - 2) \). Find the value of \( \ell \) and \( m \).

Sol:
Let \( P(x) = x^3 + \ell x^2 + mx + 24 \)
As \( (x + 4) \) is a factor of \( P(x) \),
So remainder will be zero, i.e
\[ R = P(-4) = 0 \]
\[ P(-4) = 0 \]
\[ (-4)^3 + \ell(-4)^2 + m(-4) + 24 = 0 \]
\[ -64 + 16\ell - 4m + 24 = 0 \]
\[ 16\ell - 4m - 40 = 0 \]
\[ 16\ell - 4m = 40 \]
Dividing by 4
\[ 4\ell - m = 10 \ldots \ldots (i) \]
Now as given that \( P(x) \) is divided by \( (x - 2) \) leaves a remainder 36, so
\[ R = 36 \]
i.e. \[ P(2) = 36 \]
\[ (2)^3 + \ell(2)^2 + m(2) + 24 = 36 \]
\[ 8 + 4\ell + 2m + 24 = 36 \]
\[ 4\ell + 2m + 32 = 36 \]
\[ 4\ell + 2m = 40 \]
Dividing by 2
\[ 2\ell + m = 2 \ldots \ldots (ii) \]
Adding (i) and (ii)
\[ 4\ell - m = 10 \]
\[ 2\ell + m = 2 \]
\[ 6\ell = 12 \]
\[ \ell = \frac{12}{6} \]
\[ \ell = 2 \]
Putting value of \( \ell \) in (ii)
\[ 2\ell + m = 2 \]
\[ 2(2) + m = 2 \]
\[ m = 2 - 4 \]
\[ m = -2 \]

Q.8. The expression \( \ell x^3 + mx^2 - 4 \) leaves remainder of -3 and 12 when divided by \( (x - 1) \) and \( (x + 2) \) respectively. Calculate the values of \( \ell \) and \( m \).

Sol:
Let \( P(x) = \ell x^3 + mx^2 - 4 \)
As given that \( P(x) \) when divided by \( x - 1 \) leaves remainder -3, so
\[ R = -3 \]
\[ P(1) = -3 \]
\[ \ell(1)^3 + m(1)^2 - 4 = -3 \]
\[ \ell + m - 4 = -3 \]
\[ \ell + m = 4 - 3 \]
\[ \ell + m = 1 \ldots \ldots (i) \]
As given that \( P(x) \) when divided by \( (x + 2) \) leaves the remainder 12, so
\[ R = 12 \]
\[ P(-2) = 12 \]
\[ \ell(-2)^3 + m(-2)^2 - 4 = 12 \]
\[ -8\ell + 4m - 4 = 12 \]
\[ -8\ell + 4m = 12 + 4 \]
\[ -8\ell + 4m = 16 \]
Dividing by 4
\[-2\ell + m = 4\]  
Subtracting (ii) from (i)
\[\ell + m = 1\]
\[-2\ell + m = 4\]
\[\begin{array}{c}
\ell + m = 1 \\
-2\ell + m = 4 \\
\hline
3\ell = -3 \\
\ell = \frac{-3}{3} \\
\ell = -1
\end{array}\]
Putting value of \(\ell\) in (i)
\[\ell + m = 1\]
\[-1 + m = 1\]
\[m = 1 + 1\]
\[m = 2\]

Q.9 The expression \(ax^3 - 9x^2 + bx + 3a\)
is exactly divisible by \(x^2 - 5x + 6\). Find the values of \(a\) and \(b\)

Sol:
Let \(P(x) = ax^3 - 9x^2 + bx + 3a\)
Taking \(x^2 - 5x + 6\)
\[= x^2 - 2x - 3x + 6\]
\[= x(x - 2) - 3(x - 2)\]
\[= (x - 2)(x - 3)\]
As given that \(P(x)\) is exactly divisible by \(x - 2\), so \(P(2) = 0\)
\[a(2)^3 - 9(2)^2 + b(2) + 3a = 0\]
\[8a - 36 + 2b + 3a = 0\]
\[11a + 2b = 36\]  
As given that \(P(x)\) is exactly divisible by \(x - 3\), so
\[P(3) = 0\]
\[a(3)^3 - 9(3)^2 + b(3) + 3a = 0\]
\[27a - 81 + 3b + 3a = 0\]
\[30a + 3b = 81\]
Dividing by 3
\[10a + b = 27\]  
Multiplying (ii) by 2 and subtracting (i) from it.
\[20a + 2b = 54\]
\[11a + 2b = 36\]
\[\begin{array}{c}
9a = 18 \\
a = 2
\end{array}\]
Putting value of \(a\) in (ii)
\[10a + b = 27\]
\[10(2) + b = 27\]
\[b = 27 - 20\]
\[b = 7\]

**Rational Root Theorem**
Let
\[a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n = 0, \quad a_0 \neq 0\]
be a polynomial equation of degree \(n\) with integral coefficients. If \(p/q\) is a rational root (expressed in lowest terms) of the equation, then \(p\) is a factor of the constant term \(a_n\) and \(q\) is a factor of the leading coefficient \(a_0\).

**Example**
Factorize the polynomial
\[x^3 - 4x^2 + x + 6\], by using Factor Theorem.

**Solution:**
We have \(P(x) = x^3 - 4x^2 + x + 6\).
Possible factors of the constant term \( p = 6 \) are \( \pm 1, \pm 2, \pm 3, \) and \( \pm 6 \) and of leading coefficient \( q = 1 \) are \( \pm 1 \). Thus the expected zeros (or roots) of \( P(x) = 0 \) are
\[
\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6. \text{ If } x = a \text{ is a zero of } P(x), \text{ then } (x-a) \text{ will be a factor.}
\]
We use the hit and trial method to find zeros of \( P(x) \). Let us try \( x = 1 \).
Now
\[
P(1) = (1)^3 - 4(1)^2 + 1 + 6
= 1 - 4 + 1 + 6
= 4 \neq 0
\]
Hence \( x = 1 \) is not a zero of \( P(x) \).
Again \( P(-1) = (-1)^3 - 4(-1)^2 - 1 + 6 \)
\[
= -1 - 4 - 1 + 6 = 0
\]
Hence \( x = -1 \) is a zero of \( P(x) \) and therefore,
\[
x - (-1) = (x+1) \text{ is a factor of } P(x).
\]
Now \( P(2) = (2)^3 - 4(2)^2 + 2 + 6 \)
\[
= 8 - 16 + 2 + 6 = 0 \Rightarrow x = 2 \text{ is a root.}
\]
Hence \( (x-2) \) is also a factor of \( P(x) \).
Similarly \( P(3) = (3)^3 - 4(3)^2 + 3 + 6 \)
\[
= 27 - 36 + 3 + 6 = 0 \Rightarrow x = 3 \text{ is a zero of } P(x).
\]
Hence \( (x-3) \) is the third factor of \( P(x) \).

Thus the factorized form of
\[
P(x) = x^3 - 4x^2 + x + 6 \text{ is}
\]
\[
(x+1)(x-2)(x-3).
\]

Exercise 5.4

Factorize each of the following cubic polynomials by factor theorem.

Q.1 \( x^3 - 2x^2 - x + 2 \)

Let \( P(x) = x^3 - 2x^2 - x + 2 \)

Put \( x = 1 \)
\[
P(1) = (1)^3 - 2(1)^2 - (1) + 2
= 1 - 2 - 1 + 2
= -3 + 3 = 0
\]
As, \( R = 0 \),
So \( (x-1) \) is a factor

Put \( x = -1 \)
\[
P(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2
= -1 - 2 + 1 + 2
\]
As \( R = 0 \),
So \( (x+1) \) is the second factor of \( p(x) \).

Hence \( x = -1 \) is a zero of \( P(x) \) and therefore,
\[
x - (-1) = (x+1) \text{ is a factor of } P(x).
\]
Now \( P(2) = (2)^3 - 4(2)^2 + 2 + 6 \)
\[
= 8 - 16 + 2 + 6 = 0 \Rightarrow x = 2 \text{ is a root.}
\]
Hence \( (x-2) \) is also a factor of \( P(x) \).

Similarily \( P(3) = (3)^3 - 4(3)^2 + 3 + 6 \)
\[
= 27 - 36 + 3 + 6 = 0 \Rightarrow x = 3 \text{ is a zero of } P(x).
\]
Hence \( (x-3) \) is the third factor of \( P(x) \).

Thus the factorized form of
\[
P(x) = x^3 - 4x^2 + x + 6 \text{ is}
\]
\[
(x+1)(x-2)(x-3).
\]

Q.1 \( x^3 - 2x^2 - x + 2 \)

Let \( P(x) = x^3 - 2x^2 - x + 2 \)

Put \( x = 1 \)
\[
P(1) = (1)^3 - 2(1)^2 - (1) + 2
= 1 - 2 - 1 + 2
= -3 + 3 = 0
\]
As, \( R = 0 \),
So \( (x-1) \) is the third factor

Put \( x = -1 \)
\[
P(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2
= -1 - 2 + 1 + 2
\]
As \( R = 0 \),
So \( (x+1) \) is the second factor of \( p(x) \).

Put \( x = 2 \)
\[
P(2) = (2)^3 - 2(2)^2 - (2) + 2
= 8 - 8 - 2 + 2
= 10 - 10
= 0
\]
As \( R = 0 \),
So \( (x-2) \) is the third factor

Hence \( P(x) = x^3 - 2x^2 - x + 2 \)
\[
= (x-1)(x+1)(x-2)
\]

Q.2 \( x^3 - x^2 - 22x + 40 \)

Sol:

Let \( P(x) = x^3 - x^2 - 22x + 40 \)

Put \( x = 1 \)
\[
P(1) = (1)^3 - (1)^2 - 22(1) + 40
= 1 - 1 - 22 + 40
= 1 - 22 + 40
\]
\[
=18 \neq 0
\]
Hence \( x = -1 \) is not a zero of \( P(x) \)

Put \( x = -1 \)

\[
P(-1) = (-1)^3 - (-1)^2 - 22(-1) + 40
= -1 - 1 + 22 + 40
= 60 \neq 0
\]
Hence \( x = -1 \) is not a zero of \( P(x) \)

Put \( x = 2 \)

\[
P(2) = (2)^3 - (2)^2 - 22(2) + 40
= 8 - 4 - 44 + 40 = 0
\]
Hence \( x = 2 \) is a zero of \( P(x) \)

So \( (x - 2) \) is a factor

Put \( x = -2 \)

\[
P(-2) = (-2)^3 - (-2)^2 - 22(-2) + 40
= -8 - 4 + 44 + 40 = 72
\]
Hence \( x = -2 \) is not a zero of \( P(x) \)

Put \( x = 3 \)

\[
P(3) = (3)^3 - (3)^2 - 22(3) + 40
= 27 - 9 - 66 + 40
= 67 - 75
= -8 \neq 0
\]
Hence \( x = 3 \) is not a zero of \( P(x) \)

Put \( x = -3 \)

\[
P(-3) = (-3)^3 - (-3)^2 - 22(-3) + 40
= -27 - 9 + 66 + 40
= 106 - 36
= 70 \neq 0
\]
Hence \( x = -3 \) is not a zero of \( P(x) \)

Put \( x = 4 \)

\[
P(4) = (4)^3 - (4)^2 - 22(4) + 40
= 64 - 16 - 88 + 40
\]

\[
=104 - 104
= 0
\]
Hence \( x = 4 \) is a zero of \( P(x) \)

So \( (x - 4) \) is a second factor

Put \( x = 4 \)

\[
P(-4) = (-4)^3 - (-4)^2 - 22(-4) + 40
= -64 - 16 + 88 + 40
= -80 + 128
= 48 \neq 0
\]
So, \( x = -4 \) is not a zero of \( P(x) \)

Put \( x = 5 \)

\[
P(5) = (5)^3 - (5)^2 - 22(5) + 40
= 125 - 25 - 110 + 40
= 165 - 135
= 30 \neq 0
\]
So, \( x = 5 \) is not a zero of \( P(x) \)

Put \( x = -5 \)

\[
P(-5) = (-5)^3 - (-5)^2 - 22(-5) + 40
= -125 - 25 + 110 + 40
= -150 + 150
= 0
\]
So, \( x = -5 \) is a zero of \( P(x) \)

Hence \( x + 5 \) is third factor of \( P(x) \)

Hence \( P(x) = x^3 - x^2 - 22x + 40 \)

\[
=(x - 2)(x - 4)(x + 5)
\]

**Q.3** \( x^3 - 6x^2 + 3x + 10 \)

**Sol:**

Let \( P(x) = x^3 - 6x^2 - 6x^2 + 3x + 10 \)

Put \( x = 1 \)

\[
P(1) = (1)^3 - 6(1)^2 + 3(1) + 10
= 1 - 6 + 3 + 10
\]
\[=14 - 6
\]
\[=8 \neq 0
\]
So, \(x = 1\) is not a zero of \(P(x)\)
Put \(x = -1\)
\[P(-1)=(−1)^3 − 6(−1)^2 + 3(−1)+10
\]
\[=−1 − 6 − 3 + 10
\]
\[=−19+10
\]
\[=0
\]
So, \(x = -1\) is a zero of \(P(x)\).
Hence \((x + 1)\) is a factor of \(P(x)\)
Put \(x = 2\)
\[P(2)=(2)^3 − 6(2)^2 + 3(2)+10
\]
\[=8−24+6+10
\]
\[=−24−6+10
\]
\[=0
\]
So, \(x = 2\) is a zero of \(P(x)\).
Hence \((x − 2)\) is second factor of \(P(x)\)
Put \(x = 4\)
\[P(4)=(4)^3 − 6(4)^2 + 3(4)+10
\]
\[=64−6(16)+12+10
\]
\[=86−96
\]
\[=−10 \neq 0
\]
So, \(x = 4\) is not a zero of \(P(x)\)
Put \(x = -4\)
\[P(-4)=(−4)^3 − 6(−4)^2 + 3(−4)+10
\]
\[=−64−6(16)−12+10
\]
\[=−64−96−12+10
\]
\[=−172+10
\]
\[=−162
\]
\[=−162 \neq 0
\]
Put \(x = 5\)
\[P(5)=(5)^3 − 6(5)^2 + 3(5)+10
\]
\[=125−150+15+10
\]
\[=150−150
\]
\[=0
\]
So, \(x = 5\) is a zero of \(P(x)\)
Hence \((x − 5)\) is third factor of \(P(x)\)
Hence \(P(x) = x^3 − 6x^2 + 3x + 10 = (x + 1)(x − 2)(x − 5)\)

Q. 4 \(x^3 + x^2 − 10x + 8\)

Sol:
Let \(P(x) = x^3 + x^2 − 10x + 8\)
Put \(x = 1\)
\[P(1)=(1)^3 +(1)^2 − 10(1)+8
\]
\[=1+1−10+8
\]
\[=0
\]
So, \(x = 1\) is a zero of \(P(x)\)
Hence \((x - 1)\) is a factor of \(P(x)\)

Put \(x = -1\)

\[
P(-1) = (-1)^3 + (-1)^2 - 10(-1) + 8
= -1 + 1 + 10 + 8
= 18 \neq 0
\]
So, \(x = -1\) is not a zero of \(P(x)\)

Put \(x = 2\)

\[
P(2) = (2)^3 + (2)^2 - 10(2) + 8
= 8 + 4 - 20 + 8
= 20 - 20
= 0
\]
So, \(x = 2\) is a zero of \(P(x)\)

Hence \(x - 2\) is second factor of \(P(x)\)

Put \(x = -2\)

\[
P(-2) = (-2)^3 + (-2)^2 - 10(-2) + 8
= -8 + 4 + 20 + 8
= 24 \neq 0
\]
So, \(x = -2\) is not a zero of \(P(x)\)

Put \(x = 3\)

\[
P(3) = (3)^3 + (3)^2 - 10(3) + 8
= 27 + 9 - 30 + 8
= 44 - 30
= 14 \neq 0
\]
Put \(x = -3\)

\[
P(-3) = (-3)^3 + (-3)^2 - 10(-3) + 8
= -27 + 9 + 30 + 8
= 20 \neq 0
\]
So, \(x = -3\) is not a zero of \(P(x)\)

Put \(x = 4\)

\[
P(4) = (4)^3 + (4)^2 - 10(4) + 8
= 64 + 16 - 40 + 8
= 88 - 40
= 48 \neq 0
\]
So, \(x = 4\) is not a zero of \(P(x)\)

Put \(x = -4\)

\[
P(-4) = (-4)^3 + (-4)^2 - 10(-4) + 8
= -64 + 16 + 40 + 8
= -64 + 64
= 0
\]
So, \(x = -4\) is a zero of \(P(x)\)

Hence \(x + 4\) is third factor of \(P(x)\)

Hence \(P(x) = x^3 + x^2 - 10x + 8\)

\[
= (x - 1)(x - 2)(x + 4)
\]

Q.5 \(x^3 - 2x^2 - 5x + 6\)

Sol:

\[
P(x) = x^3 - 2x^2 - 5x + 6
\]

Put \(x = 1\)

\[
P(1) = (1)^3 - 2(1)^2 - 5(1) + 6
= 1 - 2 - 5 + 6
= 7 - 7
= 0
\]
So, \(x = 1\) is a zero of \(P(1)\)

Hence \(x - 1\) is a factor of \(P(x)\)

Put \(x = -1\)

\[
P(-1) = (-1)^3 - 2(-1)^2 - 5(-1) + 6
= -1 - 2 + 5 + 6
= -3 + 11
= 8 \neq 0
\]
So, \(x = -1\) is not a zero of \(P(x)\)

Put \(x = 2\)

\[
P(2) = (2)^3 - 2(2)^2 - 5(2) + 6
\]
=8 – 8 + 6
= 4 ≠ 0
So, x = 2 is not a zero of P(x)
Put x = -2
P(-2) = (-2)^3 - 2(-2)^2 - 5(-2) = -8 - 8 + 10 + 6 = 0
So, x = -2 is a zero of P(x)
Hence (x + 2) is second factor of P(x)
Put x = 3
P(3) = (3)^3 - 2(3)^2 - 5(3) + 6 = 27 - 18 - 15 + 6 = 33 - 33 = 0
So, x = 3 is a zero of P(x)
Hence (x - 3) is third factor of P(x)
Hence P(x) = x^3 - 2x^2 - 5x + 6
= (x - 1)(x + 2)(x - 3)
Q.6 x^3 + 5x^2 - 2x - 24
Sol:
Let P(x) = x^3 + 5x^2 - 2x - 24
Put x = 1
P(1) = (1)^3 + 5(1)^2 - 2(1) - 24 = 1 + 5 - 2 - 24 = 6 - 26 = -20 ≠ 0
So, x = 1 is not a zero of P(x)
Put x = -1
P(-1) = (-1)^3 + 5(-1)^2 - 2(-1) - 24 = -1 + 5 + 2 - 24 = 7 - 25
= -18 ≠ 0
So, x = -1 is not a zero of P(x)
Put x = 2
P(2) = (2)^3 + 5(2)^2 - 2(2) - 24 = 8 + 20 - 4 - 24 = 28 - 28 = 0
So, x = 2 is a zero of P(x)
Hence (x - 2) is a factor of P(x)
Put x = -2
P(-2) = (-2)^3 + 5(-2)^2 - 2(-2) - 24 = -8 + 5(4) + 4 - 24 = -32 + 24 = -8 ≠ 0
So, x = -2 is not a zero of P(x)
Put x = 3
P(3) = (3)^3 + 5(3)^2 - 2(3) - 24 = 27 + 5(9) - 6 - 24 = 72 - 30 = 42 ≠ 0
So, x = 3 is not a zero of P(x)
Put x = -3
P(-3) = (-3)^3 + 5(-3)^2 - 2(-3) - 24 = -27 + 45 + 6 - 24 = 51 - 51 = 0
So, x = -3 is a zero of P(x)
Hence (x + 3) is second factor of P(x)
Put x = 4
P(4) = (4)^3 + 5(4)^2 - 2(4) - 24 = 64 + 5(16) - 8 - 24 = 144 - 32
\[ =112 \neq 0 \]
So, \( x = 4 \) is not a zero of \( P(x) \)
Put \( x = -4 \)
\[
P(-4) = (-4)^3 + 5(-4)^2 - 2(-4) - 24
\]
\[
= -64 + 80 + 8 - 24
\]
\[
= 0
\]
So, \( x = -4 \) is a zero of \( P(x) \)
Hence \((x + 4)\) is third factor of \( P(x) \)
Hence \( P(x) = x^3 + 5x^2 - 2x - 24 \)
\[
= (x - 2)(x + 3)(x + 4)
\]
Q. 7 \( 3x^3 - x^2 - 12x + 4 \)
Sol: \( P(x) = 3x^3 - x^2 - 12x + 4 \)
Put \( x = 1 \)
\[
P(1) = 3(1)^3 - (1)^2 - 12(1) + 4
\]
\[
= 3 - 1 - 12 + 4
\]
\[
= 7 - 13
\]
\[
= -6 \neq 0
\]
So, \( x = 1 \) is not a zero of \( P(x) \)
Put \( x = -1 \)
\[
P(-1) = 3(-1)^3 - (-1)^2 - 12(-1) + 4
\]
\[
= -3 - 1 + 12 + 4
\]
\[
= 9 + 16
\]
\[
= 25 \neq 0
\]
So, \( x = -1 \) is not a zero of \( P(x) \)
Put \( x = 2 \)
\[
P(2) = 3(2)^3 - (2)^2 - 12(2) + 4
\]
\[
= 24 - 4 - 24 + 4
\]
\[
= 28 - 28
\]
\[
= 0
\]
So, \( x = 2 \) is a zero of \( P(x) \)
Hence \((x - 2)\) is a factor of \( P(x) \)
Put \( x = -2 \)
\[
P(-2) = 3(-2)^3 - (-2)^2 - 12(-2) + 4
\]
\[
= -24 + 4 + 24 + 4
\]
\[
= -28 + 28
\]
\[
= 0
\]
So, \( x = -2 \) is a zero of \( P(x) \)
Hence \((x + 2)\) is second factor of \( P(x) \)
Put \( 3x = 1 \)
\[
x = \frac{1}{3}
\]
\[
P \left( \frac{1}{3} \right) = 3 \left( \frac{1}{3} \right)^3 - \left( \frac{1}{3} \right)^2 - 12 \left( \frac{1}{3} \right) + 4
\]
\[
= \frac{1}{27} - \frac{1}{9} - 12 \left( \frac{1}{3} \right) + 4
\]
\[
= \frac{1}{9} - \frac{1}{9} - 4 + 4
\]
\[
= 0
\]
So, \( x = \frac{1}{3} \) is a zero of \( P(x) \)
Hence \((3x - 1)\) is third factor of \( P(x) \)
Hence \( P(x) = 3x^3 - x^2 - 12x + 4 \)
\[
= (x - 2)(x + 2)(3x - 1)
\]
Q. 8 \( 2x^3 + x^2 - 2x - 1 \)
Let \( P(x) = 2x^3 + x^2 - 2x - 1 \)
Put \( x = 1 \)
\[
P(1) = 2(1)^3 + (1)^2 - 2(1) - 1
\]
\[
= 2 + 1 - 2 - 1
\]
\[
= 3 - 3
\]
\[
= 0
\]
So, \( x = 1 \) is a zero of \( P(x) \)
*Hence \((x - 1)\) is a factor of \( P(x) \)
Put \( x = -1 \)
\[ P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 \]
\[ = -2 + 1 + 2 - 1 \]
\[ = -1 + 1 \]
\[ = 0 \]
So, \( x = -1 \) is a zero of \( P(x) \)

Hence \((x + 1)\) is second factor of \( P(x) \)

Put \( 2x = 1 \)
\[ x = \frac{1}{2} \]
\[ P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) - 1 \]
\[ = 2\left(\frac{1}{8}\right) + \frac{1}{4} - 1 \]
\[ = \frac{1}{4} + \frac{1}{4} - 1 \]
\[ = -\frac{3}{2} \neq 0 \]
So, \( x = \frac{1}{2} \) is not a zero of \( P(x) \)

Put \( x = -\frac{1}{2} \)
\[ P\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) - 1 \]
\[ = 2\left(-\frac{1}{8}\right) + \frac{1}{4} + 1 - 1 \]
\[ = -\frac{1}{4} + 1 - 1 \]
\[ = 0 \]
So, \( x = -\frac{1}{2} \) is a zero of \( P(x) \)

Hence \( 2x + 1 \) is third factor of \( P(x) \)

Hence \( P(x) = 2x^3 + x^2 - 2x - 1 \)
\[ = (x - 1)(x + 1)(2x + 1) \]

### Objective

1. The factor of \( x^2 - 5x + 6 \) are:____
   - (a) \( x + 1, x - 6 \)
   - (b) \( x - 2, x - 3 \)
   - (c) \( x + 6, x - 1 \)
   - (d) \( x + 2, x + 3 \)

2. Factors of \( 8x^3 + 27y^3 \) are:____
   - (a) \((2x+3y)(4x^2-y^2)\)
   - (b) \((2x-3y)(4x^2+9y^2)\)
   - (c) \((2x+3y)(4x^2-6xy+y^2)\)
   - (d) \((2x-3y)(4x^2+6xy+y^2)\)

3. Factors of \( 3x^2 - x - 2 \) are:
   - (a) \((x+1)(3x-2)\)
   - (b) \((x+1)(3x+2)\)
   - (c) \((x-1)(3x-2)\)
   - (d) \((x-1)(3x+2)\)

4. Factors of \( a^4 - 4b^4 \) are:____
   - (a) \((a-b)(a+b)(a^2+4b^2)\)
   - (b) \((a^2-2b^2)(a^2+2b^2)\)
   - (c) \((a-b)(a+b)(a^2-4b^2)\)
   - (d) \((a^2-2b^2)(a^2-2b^2)\)

5. What will be added to complete the square of \( 9a^2 - 12ab \)?____
   - (a) \(-16b^2\)
   - (b) \(16b^2\)
   - (c) \(4b^2\)
   - (d) \(-4b^2\)

6. Find \( m \) so that \( x^2 + 4x + m \) is a complete square:
   - (a) \(8\)
   - (b) \(-8\)
   - (c) \(4\)
   - (d) \(16\)

7. Factors of \( 5x^2 - 17xy - 12y^2 \) are____
   - (a) \((x+4y)(5x+3y)\)
   - (b) \((x-4y)(5x-3y)\)
   - (c) \((x-4y)(5x+3y)\)
   - (d) \((5x-4y)(x+3y)\)
8. Factors of $27x^3 - \frac{1}{x^3}$ are

(a) $\left(3x - \frac{1}{x}\right) \left(9x^2 + 3 + \frac{1}{x^2}\right)$
(b) $\left(3x + \frac{1}{x}\right) \left(9x^2 + 3 + \frac{1}{x^2}\right)$
(c) $\left(3x - \frac{1}{x}\right) \left(9x^2 - 3 + \frac{1}{x^2}\right)$
(d) $\left(3x + \frac{1}{x}\right) \left(9x^2 - 3 + \frac{1}{x^2}\right)$

9. If $x - 2$ is a factor of $p(x) = x^2 + 2kx + 8$, then $K =$

(a) $-3$  (b) $3$  (c) $4$  (d) $5$

10. $4a^2 + 4ab + (\ldots)$ is a complete square

(a) $b^2$  (b) $2b$  (c) $a^2$  (d) $4b^2$

11. $\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2} =$

(a) $\left(\frac{x - y}{y x}\right)^2$  (b) $\left(\frac{x + y}{y x}\right)^2$
(c) $\left(\frac{x - y}{y x}\right)^3$  (d) $\left(\frac{x + y}{y x}\right)^3$

12. $(x+y) (x^2 - xy + y^2) =$

(a) $x^3 - y^3$  (b) $x^3 + y^3$  (c) $(x+y)^3$  (d) $(x-y)^3$

13. Factors of $x^4 - 16$ is

(a) $(x-2)^2$  (b) $(x-2)(x+2)(x^2+4)$  (c) $(x-2)(x+2)$  (d) $(x+2)^2$

14. Factors of $3x - 3a + xy - ay$.

(a) $(3+y)(x-a)$  (b) $(3-y)(x+a)$  (c) $(3-y)(x-a)$  (d) $(3+y)(x+a)$

15. Factors of $pqr + qr^2 - pr^2 - r^3$

(a) $r(p+r)(q-r)$  (b) $r(p-r)(q+r)$  (c) $r(p-r)(q-r)$  (d) $r(p+r)(q+r)$

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**Answer Key**

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